tive ions is, say, twice as great as that carried by the positive ions, for equal numbers of positive and negative ions are produced by the ionisation of the neutral gas.

Attempts were now made to find an answer to the first question suggested above—Is there any evidence that ions are likely to be present under normal conditions in the atmosphere ?

Former experiments furnished a certain amount of evidence in favour of an affirmative answer.

When moist dust-free air is allowed to expand suddenly a rain-like condensation always take place if the maximum supersaturation exceeds a certain limit. This limit is identical with that required to make water condense on ions ; the identity is in fact so exact as to furnish what is at first sight almost convincing evidence that ordinary moist air is always to a very slight extent ionised. The number of these nuclei is too small to make the absence of sensible electrical conductivity in air under ordinary conditions inconsistent with the view that they are ions.

All attempts, however, to remove these nuclei, by applying a strong electric field such as would have removed ordinary ions almost as fast as they were produced, have failed, even when a differential apparatus was used, such as would have made manifest even a slight diminution in the number of the nuclei by the action of the field. The same is true of the similar nuclei produced by the action of weak ultra-violet light on moist air.

Such nuclei, therefore, in spite of their identity as condensation nuclei with the ions, cannot be regarded as free ions, unless we suppose the ionisation to be developed by the process of producing the supersaturation. This question requires further investigation.

"Data for the Problem of Evolution in Man. II. A First Study of the Inheritance of Longevity and the Selective Death-rate in Man." By Miss MARY BEETON and KARL PEARSON, F.R.S., University College, London. Received May 29,—Read June 15, 1899.

1. According to Wallace and Weismann* the duration of life in any organism is determined by natural selection. An organism lives so long as it is advantageous, not to itself, but to its species, that it should live. But it would be impossible for natural selection to determine the fit duration of life, as it would be impossible for it to fix any other character, unless that character were inherited. Accordingly the hypothesis above referred to supposes that duration of life is an

* See Weismann, 'On Heredity,' Essays I and II, and especially Professor Poulton's note as to Wallace, p. 23, of first English edition.

inherited character. So far as we are aware, however, neither the above-mentioned naturalists nor any other investigators have published researches bearing on the problem of whether duration of life is or is not inherited. We are accustomed to hear of a particular man that "he comes of a long-lived family," but the quantitative measure of the inheritance of life's duration does not yet seem to have been determined. This absence of investigation appears the more remarkable as a knowledge of the magnitude of inheritance in this respect would, we should conceive, be of primary commercial importance in the consideration of life insurance and of annuities. The biological interest of the problem, as we have already noticed, is very great.

2. It will be well in the first place to point out that the problem is by no means an easy or a straightforward one. The ages at death of even close relatives must be found from records of some kind, or else collected *ab initio*. Now if we take records like the Peerage, Baronetage, Landed Gentry, family histories, and private pedigrees, we find various serious omissions. In the first place the ages of the women are rarely given, pages of the Peerage or Landed Gentry may be examined before a single record is found of the ages of two sisters at death, or even of a mother and a daughter. Further, the census and other returns show how liable we are to find women's ages given erroneously. Family histories and pedigrees suffer in the same way, the pedigrees are mostly taken through the male line, and the women's ages can only be found in rare cases. An exception must be made in the case of the Quaker family histories, such as those of the Backhouse, Whitney, and other families. Here the data are as full for the women as for the men, but naturally the history of a single, even much-branched family, does not provide anything like the material that the Peerage and Landed Gentry do in the case of men. For this reason our first study is confined to inheritance of longevity in the male line. We hope eventually to collect enough data for inheritance in the female line, but it will take a longer time to amass, and we fear will scarcely be as homogeneous.

In the second place, the sources to which we have referred omit more or less completely all record of the ages at death of infants and children. The Quaker records give better results than the Peerage, but even here the great bulk of child deaths appears to remain unrecorded. Out of 1000 males born in this country more than 300 die before they are 20 years of age. But when 1000 cases of ages of father and son were taken out of the Landed Gentry, only 31 cases showed the death of a son before 20 years of age. Of 2000 brothers from the Peerage in 1000 pairs, only 21 individuals died before 20 years of age. In the Quaker histories we found about 16 per cent. of deaths before 20 years of age. Clearly such early deaths are not represented in anything like their proper proportions. They will have to be found from other sources; possibly by direct inquiry, and

the issue of data cards. We were thus compelled to limit this first study to the case when both relatives die at a greater age than 20. In the case of fathers, when we are dealing with the correlation between father's and son's ages at death, this is practically no limitation at all, as no father dying under 20 years of age was met with. In the case of the offspring, however, the limitation cuts off the distribution somewhat abruptly with a finite ordinate at 20—25, five years being our unit of grouping.

3. Now duration of life is a very different character to eye-colour, or to some extent to the size of organs in adult life. Eye-colour is fairly well determined, it may change with old age slightly, but it cannot transform itself from light blue to brown. Again, nourishment and use undoubtedly affect the size of organs, but they are likely to influence father and son, or, at any rate, brother and brother, in much the same way, for they are members of the same family and the same class. On the other hand, death depends not only on inherited constitution but on innumerable chance elements of environment and circumstance. The environment both of home and period is much more alike for two brothers than for a father and son; food, sanitation, habits of life, change considerably in a generation, and two brothers have more equal chances of life than a father and son. But even with two brothers, one may live on the family estates and the other ruin his health in Africa or India. Hence, while the non-differential death-rate will not materially alter the correlations between most characters in relatives, it must seriously affect the correlation between the durations of life in father and son, and to a lesser extent between brother and brother. A good stock may be better protected against death than a weak one, but no stock at all can resist certain attacks. Hence if we look upon death as a marksman, p per cent. of his shots are, we may say, sure to be effective whatever they hit, this is the non-differential death-rate, the remaining $100-p$ per cent. of his attacks will only be successful on the weaker stocks. Now the effect of this conception of death's action is that the correlation table for ages at death of any pair of relatives must be looked upon as a mixture of uncorrelated material—deaths due to the non-differential death-rate, and correlated material—deaths due to the differential or selective death-rate. At different periods of life also one of these death-rates may give more material to the table than at another. In the case of fathers and sons we should expect the non-differential death-rate to be more numerous in its contributions than in the case of brother and brother.

Now it has been shown by one of us that when correlated material is mixed with uncorrelated material, the result is approximately to reduce the coefficient of correlation in the ratio of the amount of correlated to the total amount of material.* Hence, if we assume that the

* 'Phil. Trans.,' A, vol. 192, p. 277.

actual correlation between constitutional strengths to resist death would be given, at any rate approximately, by the values determined for other characters in a memoir on the Law of Ancestral Heredity,* we have clearly a method of to some extent ascertaining the proportion of the selective and non-selective death-rates in man. In the sequel it will be shown that from the age of 20 to the end of life our tables give a correlation between the duration of life of father and son of about 0·12 to 0·14, and between brother and brother of about 0·26. According to the Law of Ancestral Heredity we should expect these quantities to be about 0·3 and 0·4. Hence we conclude that the amounts of correlated material in the two cases are 40 to 50 per cent. and 65 per cent. But if pN be the number of cases in which the death-rate is selective for N individuals, p^2N will be the number of cases for which it is selective when we take pairs of individuals. In other words the selective death-rate in the first case† is about 63 to 70 per cent. and in the second about 80 per cent. of the total death-rate. Without laying great stress on the actual numbers just stated, we think that they are sufficiently close to demonstrate that a substantial selective death-rate actually exists at work on mankind, and that with like environment it may amount to as much as four times the non-selective death-rate.‡ In other words, having demonstrated that duration of life is really inherited, we have thereby demonstrated that natural selection is very sensibly effective among mankind. The natural selection we are here dealing with is not in the first place, of course, a result of any struggle of individual with individual, but of individual with environment and with the defects of personal physique.

4. In order to show the biological importance of investigating the inheritance of duration of life, we have cited the results obtained for correlation between the ages at death of father and son and brother and brother. But the method by which these results were obtained requires further discussion. We have already seen the need to exclude deaths under 20 years, but even then we have not got in the case of father and son two like groups of material. The father has been more severely selected than the son. He has lived to become a father, and he is strong enough to be the father of a son who lives to be

* 'Roy. Soc. Proc.,' vol. 62, pp. 397 and 400.

† This selective death-rate from the data for father and son must be interpreted in the sense indicated above. The drop from 80 to 65, say, per cent. is in itself a measure of the change of environment of the two generations.

‡ The correlation on which this determination is based might be illusory, if families were reared under very individual environments; the correlation in duration of life of brothers, for example, might then be a result of their individual family environment. But the environment when we take comparatively homogeneous classes like the Peerage or Landed Gentry must be very similar, and we think this source of error, suggested to us by Professor Weldon, while very real has been sufficiently provided against.

20 years of age. Evidence of this selection is to be found in the facts that (1) fathers have a mean age 5 to 7 years greater than that of their sons; (2) the variability of their age at death is very sensibly less than the variability of their sons' age, *i.e.*, as 2·9 to 3·5, or (3) by noticing for example that in our first table 82 sons as against 20 fathers die before 32·5 years of age, and that in our second table some 100 sons as against 20 fathers die before 35 years of age. Clearly the group "son" is a much weaker type than the group "father." As will be shown in a memoir on the effect of selection on correlation, this want of likeness in fathers and sons itself tends to modify the correlation between them.*

While this selection occurs only in the case of fathers and sons and not in the case of brethren, still the general character of the correlation surface is alike in both. It is known that the curve of frequency of death at different ages† is by no means normal. It is probably compound, and only approximates to normality round three score years and ten. It would hardly, however, fulfil a useful purpose to deal only with the correlation of ages of death of relatives both dying under the old age mortality group, even if on the sunny side of 70 we could distinguish old age from middle age mortality. But in dealing with correlation and regression in such cases as this, we must throw entirely on one side any notion of normal surface and curves of error, and go simply to the kernel of the affair.

What we want is the law connecting the mean age at death of one relative when another relative has died at a given age. When the given age of the latter and the mean age of the former are plotted to form a curve, this curve is the regression curve whatever be the form of the frequency surface. The line of closest fit to this curve is the regression line, and Yule's theorem‡ tells us that the slope of this line is found in exactly the same way as if the frequency surface were a normal distribution. The slope of this line has nothing whatever to do with the particular form of surface, and may be found even if we cut off a portion of the surface parallel to one axis, *e.g.*, if we take the regression line for fathers or sons we get the best fitting lines in precisely the same manner whether we take all sons dying from infancy to old age, or only those from 20 years onwards. If, of course, the regression curve is sensibly linear, then the regression line is the true curve of regression. Everything proved in the memoir, "On the Law of Ancestral Heredity"§ holds for such linear regression equally well; we need not suppose normal correlation. Now the reader has

* Not of course very largely, still with the values given in the first series of fathers and sons, the correlation would be reduced about 0·86 to 0·9 of its value by the selection of fathers.

† 'Phil. Trans.,' A, vol. 186, p. 406 and plate 16.

‡ 'Roy. Soc. Proc.,' vol. 60, p. 480.

§ 'Roy. Soc. Proc.,' vol. 62, p. 386.

only to look at our regression diagrams, in particular at that for brethren, to assure himself that no curve will serve for practical purposes substantially better than a straight line. Now, if σ_x be the standard deviation of the relative whose mean age at death is taken, and σ_y of the relative of a given age at death, and r be the correlation defined by

$$r = \mathrm{S}\{z(x-m_x)(y-m_y)\}/(\mathrm{N}\sigma_x\sigma_y),$$

where z is the frequency of deviations $x-m_x$ and $y-m_y$ from the means m_x and m_y in the total observations N, then the line of closest fit, the regression line, passes through m_x and m_y, and has $r\sigma_x/\sigma_y$ for its slope. All this is independent of any theory of frequency distribution, and the vanishing of r with the correlation simply flows from the fundamental problem that the chance of a combined event is the product of two independent probabilities. Our conclusions in this paper are deduced from the above value of r and from the slope of the regression line, and they involve no further assumption than the approximate linearity of the regression curves. Our appeal to the memoir, "On the Law of Ancestral Heredity" makes also no greater demands.

5. We now turn to the material itself. Our data consist of three series, from which all deaths recorded as accidents, an exceedingly small proportion of the whole, were excluded. In excluding these we of course slightly, but very slightly, reduced the non-selective death-rate. In the first series, 1000 cases of the ages of fathers and sons at death, the latter being over 22·5 years of age, were taken from 'Foster's Peerage'; in the second series a 1000 pairs of fathers and sons, the latter dying beyond the age of 20, were taken from 'Burke's Landed Gentry'; and in the third series the ages at death of 1000 pairs of brothers dying beyond the age of 20 were taken from the Peerage.

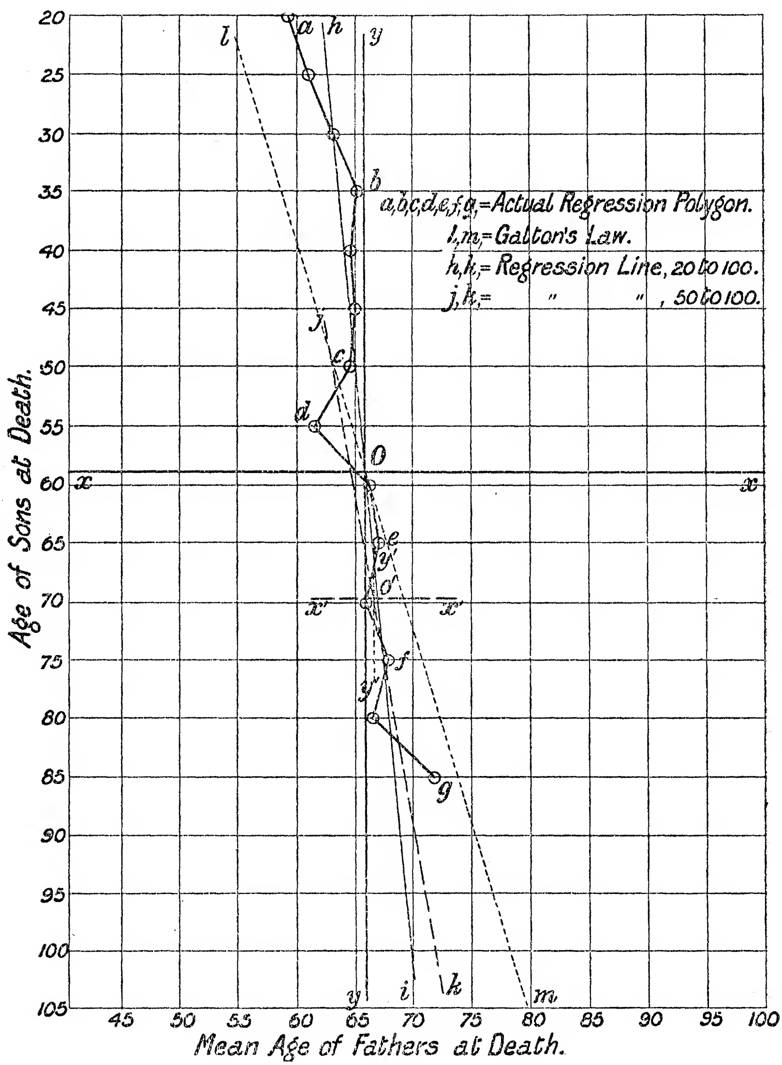
The first series was obtained by grouping all fathers dying between 22·5 and 27·5, 27·5 and 32·5, &c. We started at 22·5 because this was the earliest recorded death of a father among those extracted from the Peerage, and to have sons dying in the same range they were also started at 22·5 years. In extracting the ages at death, they were taken to the nearest whole year, and consequently in the subsequent grouping we were spared decimals. In the second and third series we originally took all deaths from birth onwards also to the nearest whole year, and then grouped in five-year periods; thus fractions were introduced when a death fell on a five-year division. Subsequently we eliminated the few deaths occurring before 20 years of age.

The aggregate material for the three series is given in Tables I, II, and III; and the means of the arrays of fathers' ages at death for sons dying at a given age, *i.e.*, the regression polygons of fathers' on sons' age at death in figs. 1 and 2; the regression polygon for brethren is given in fig. 3.

In the case of brothers, we have rendered the original distribution,

which was nearly symmetrical, absolutely symmetrical, by entering into the table each pair of brothers twice, an individual first appearing as a first brother and then as a second brother. Thus the mean age at death and variability of age at death of both sets of brothers appears the same, and we have a nominal 2000 instead of a 1000 entries. Of course in calculating the probable errors of the constants, 1000 has been taken as the number of observations. We shall now consider these diagrams and tables a little at length.

FIG. 1.—Diagram giving Mean Age of Fathers at Death for Sons dying at a given age. First Series, 1,000 cases.



6. *First Series.*—The means of the arrays of fathers for a given age at death of the son, are shown by the broken line *abcdefg* in fig. 1. The point *a* for the group of sons dying between 17·5 and 22·5 years was put in from a few observations not afterwards included in the table. Beyond the group 82·5 to 87·5 years, there were not sufficient observations to form a reliable mean at all: *yy* gives the mean age of all the 1000 fathers observed, and represents 65·835 years, *xx* gives the mean age of the 1000 sons, and represents 58·775 years. The former may be taken as the mean age at death of all fathers, the latter was only the mean age at the death of sons who live more than 22·5 years. The regression curve is a somewhat broken polygon, but one or two points may be deduced at once from it.

(*a*) It is entirely to the left of *yy* above *xx* and entirely to the right of *yy* below *xx*. Thus there is certainly correlation between the ages at death of father and son. A son dying below the mean age will have on the average a father dying below the mean age, and a son dying above the mean age will have on the average a father dying above the mean age. Graphically we see that correlation must exist. The straight line which best fits the regression polygon is given on the diagram by *hi*. The Law of Ancestral Heredity would give *lm* with a slope of 0·3. It is clear that with a quite sensible regression there is a quite sensible divergence from the law of inheritance, in other words, the death-rate is only in part selective.

Quite similar results are to be observed in fig. 2; there is again a very sensible correlation, but it is sensibly less than that required by the Law of Ancestral Heredity. The lines are lettered the same. Numerically, if M_S, M_F be the mean ages at death of sons and fathers, σ_S, σ_F their standard deviations, r_{SF} their correlation, $R_{SF} = r_{SF}\,\sigma_S/SF$, $R_{FS} = r_{SF}\,\sigma_F/\sigma_S$ the regression coefficients of son on father and father on son, we have—

First Series.		Second Series.
'Peerage,' Fathers and sons, 25 years and on.		'Landed Gentry,' Fathers and sons, 20 years and on.
65·835 years	M_F	65·9625 years
58·775 ,,	M_S	60·9150 ,,
14·6382 ,,	σ_F	14·4308 ,,
17·0872 ,,	σ_S	17·0986 ,,
0·1149 ± 0·0210	r_{SF}	0·1418 ± 0·0209
0·0985 ± 0·0182	R_{FS}	0·1196 ± 0·0178
0·1341 ± 0·0367	R_{SF}	0·1682 ± 0·0371

Now these results extracted from very different records are in good accordance. The values of the correlation and regressions are 5 to 7

times the magnitudes of their probable errors, and they agree within the probable error of their differences. The only significant difference is the mean age of deaths of sons in the Landed Gentry, which is some two years higher than in the Peerage. This is the more noteworthy in that we have begun our peerage record at 25 and not 20. Clearly the sons of the Landed Gentry are longer lived. We have undoubtedly correlation, say somewhere about 0·12, sensible and definite in amount, but clearly considerably below the 0·3 required by the law of inheritance.

(*b*) A second point may be noticed by looking at the diagrams (1) and (2), namely, that from about the age of 32·5 to 52·5 the regression line is sensibly vertical, or when the son dies in middle life, the mean age of death of the father is sensibly uncorrelated with it. In other words, we have the remarkable result that the mortality which in a paper on skew variation by one of us,* has been termed that of middle life is largely uninherited. It is during this period of life that the non-selective death-rate is chiefly predominant. After this period the regression curve becomes sensibly steeper, although not fully up to the steepness of the line given by Galton's Law. This is more properly the inheritance of *longevity*. The inheritance of duration of life may not be continuous.

If we seek the best fitting straight line for the regression polygon from 50 years onward we find :—

First Series.		Second Series.
'Peerage,' 52·5 years of son and on.		'Landed Gentry,' 50 years of son and on.
66·680 years	M_F	66·878 years
69·686 ,,	M_S	68·960 ,,
14·6734 ,,	σ_F	14·3273 ,,
9·6148 ,,	σ_S	10·4055 ,,
0·1156 ± 0·0232	r_{FS}	0·1125 ± 0·0243
0·1764 ± 0·0380	R_{FS}	0·1549 ± 0·0333

Results such as these are as close as we could expect, and they mark an increase in the steepness of the regression line from about 0·11 to 0·17, an undoubtedly substantial increase of the selective death-rate as we approach old age. The regression line for this old age mortality is marked as *jk* in diagrams (1) and (2), and we see the advance towards the Galtonian value.

(*c*) Below 32·5 years the regression line in figs. 1 and 2, especially the former, seems to indicate increased correlation again, but unfortu-

* 'Phil. Trans.,' A, vol. 186, p. 408, and Plate XVI.

nately our records do not give enough data to determine its form in a reliable manner.

Fig. 1 seems to indicate a great approach to the Galtonian value towards youth, and we should not be surprised to find the selective death-rate in youth and infancy even more predominant than in old age. This would be the inheritance of the reverse of longevity, of "brachybioty." The regression curve for this portion of life cannot be determined from our present statistics, but we hope to return to it in a second study when more data have been collected.* So far as we are able to judge at present the inheritance of the duration of life breaks up into two parts, an inheritance telling its tale in youth and another after middle life. It is the former part which seems to us to have most bearing on the fertility and survival of stocks, most individuals having reproduced themselves by 50 years of age. It is the latter part only, the true inheritance of longevity, to which it would appear that Weismann and Wallace's arguments apply :—†

"For it is evident than when one or more individuals have provided a sufficient number of successors, they themselves, as consumers of nourishment in a constantly increasing degree, are an injury to those successors. Natural selection therefore weeds them out, and in many cases favours such races as die almost immediately after they have left successors."

7. We now turn to the third series, giving the correlation between the ages of death of brothers. The data give the following numerical results :—

$$\begin{aligned} M_B &= 60{\cdot}971 \text{ years.} \\ \sigma_B &= 16{\cdot}8354 \text{ ,,} \\ r_{BB} &= 0{\cdot}2602 \pm 0{\cdot}0199. \\ R_{BB} &= 0{\cdot}2602 \pm 0{\cdot}0216. \end{aligned}$$

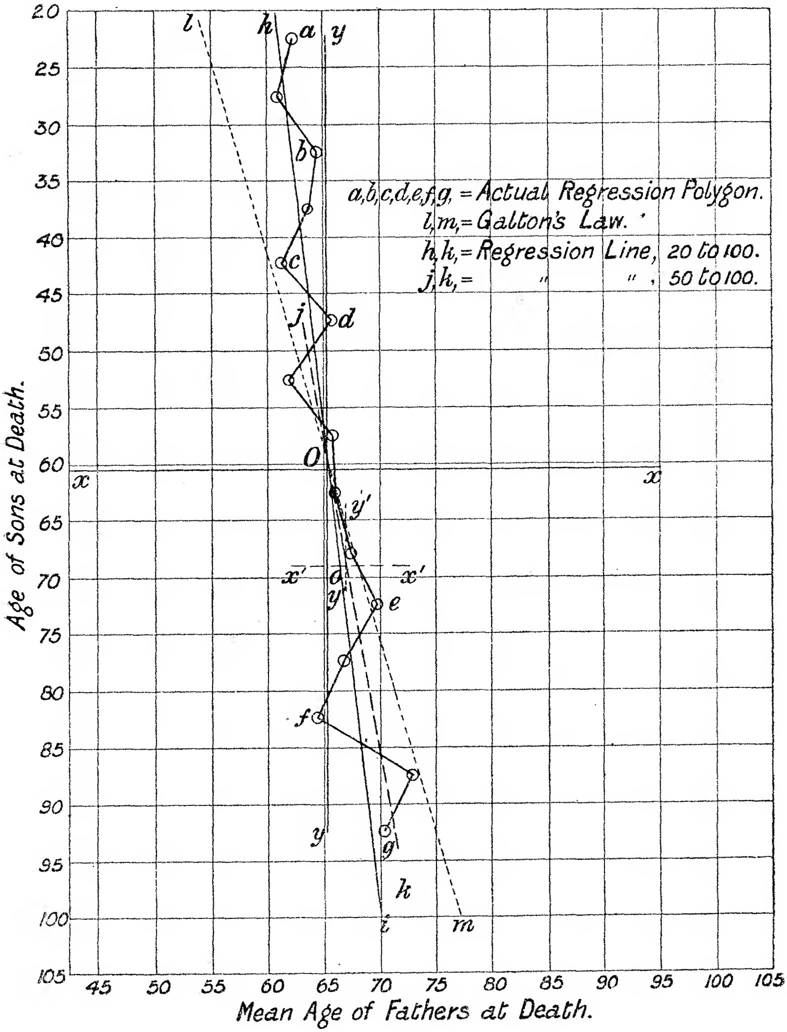
The results here are in good agreement with those for sons in the Landed Gentry, *i.e.*, in the second series. The mean age of one of a pair of brothers is slightly greater and the variability of one who has a brother slightly less than in the case of sons. But this is exactly what we might expect, considering that "brothers" are a selection from "sons," and a brother is likely to have greater vitality than a son. The group sons covers sons of fathers who did not live to have more than *one* son, and who therefore came of any early dying stock, while brothers denotes at least two sons, and therefore on the average some years more life than is necessary for one son.

The values of the coefficients of correlation and regression are some 13 times their probable errors, and we have a substantial correlation,

* This collection has already commenced, and we hope shortly to give more definite information on this point.

† Wallace, *loc. cit.*, *supra.*

FIG. 2.—Diagram giving Mean Age of Fathers at Death for Sons dying at a given Age. Second Series, 1000 cases.



approaching much closer than in the case of sons to the value demanded (0·4) by the Law of Ancestral Heredity. The diagram shows (i) how substantial is the correlation; (ii) how much more nearly the regression line kk given by observation approaches the theoretical line lm; and (iii) how very nearly the regression curve is truly linear. The reason of this closer approach to the theoretical value of heredity is owing to the diminution in the non-selective death-rate, the environ

ment of brothers during their lives being as a rule much more alike than that of father and son. It must be noted that the predominance of the non-selective death-rate in middle life, so marked in the latter case, no longer appears in the case of brothers. This would suggest that the environments of father and son differ most in middle life and are then much more unlike than those of brothers.

8. We conclude this first study by putting on record formulæ for estimating the age at death of a man, using the theory of multiple correlation as developed in a memoir* by one of the present writers, and taking as basis the second and third series, which seem to us to present the best results.

Let P be the probable age in years at death of a man, F be the age at death of his father, S_1 of his first son, S_2 of his second son, B_1 of his first brother, B_2 of his second brother. Then we have the following cases :—

Prediction of Age at Death. All Deaths after 20 Years.

(*a*) From age of father at death—

$$P = 49{\cdot}8201 + 0{\cdot}1682\ F, \qquad \Sigma = 16{\cdot}9259.$$

(*b*) From age of brother at death—

$$P = 45{\cdot}1063 + 0{\cdot}2602\ B_1, \qquad \Sigma = 16{\cdot}2555.$$

(*c*) From age of son at death—

$$P = 58{\cdot}6771 + 0{\cdot}1196\ S_1, \qquad \Sigma = 14{\cdot}2850.$$

(*d*) From ages of father and brother at death—

$$P = 37{\cdot}6647 + 0{\cdot}12685\ F + 0{\cdot}24502\ B_1, \qquad \Sigma = 16{\cdot}4099.$$

(*e*) From ages of father and son at death—

$$P = 48{\cdot}7991 + 0{\cdot}15706\ F + 0{\cdot}11168\ S, \qquad \Sigma = 14{\cdot}1573.$$

(*f*) From ages of two brothers at death—

$$P = 35{\cdot}7930 + 0{\cdot}206475\ (B_1 + B_2), \qquad \Sigma = 15{\cdot}9052.$$

(*g*) From ages of two sons at death—

$$P = 54{\cdot}3928 + 0{\cdot}09497\ (S_1 + S_2), \qquad \Sigma = 14{\cdot}1987.$$

(*h*) From ages of brother and son at death—

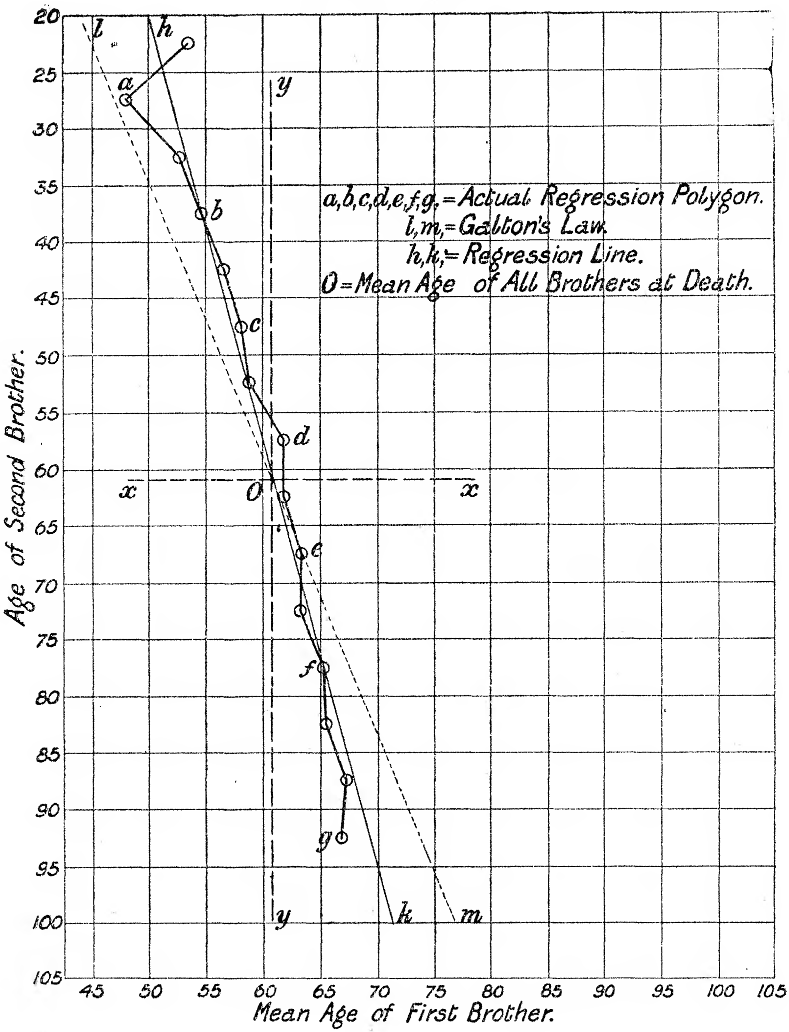
$$P = 44{\cdot}2601 + 0{\cdot}1046\ S + 0{\cdot}2514\ B, \qquad \Sigma = 13{\cdot}8508.$$

Here Σ is the standard deviation of the array of men for each group. Such formulæ† seem to us to give a quantitative accuracy to much

* "Contributions to the Mathematical Theory of Evolution. III. Regression, Heredity, and Panmixia," 'Phil. Trans.,' A, vol. 187, pp. 253—318.

† In obtaining the formulæ for prediction from the age at death of *two* relatives, certain assumptions have had to be made. Thus the correlation of ages of a man and his grandfather and of a man and his uncle at death, being at present unknown, were taken to be half the correlation of father and son. This cannot be far wrong, but the actual values ought to be found. We did not feel justified in assuming the

Fig. 3.—Diagram giving the Mean Age of Man at Death for a Brother dying at a given Age. 1000 cases.



that is allowed rather indefinite weight at present in the actuarial and medical professions. Based on a wider mass of data and a larger series of relationships we cannot but believe they would be of much help to

variability of grandfathers, which must be less than that of fathers, or their mean age at death, which must be greater than that of fathers, in order to determine the probable age at death of a man from that, say, of his grandfather and father, which would be of much interest. We only wish to draw attention to what we believe to be a new and important field of enquiry, and to indicate the nature of its problems.

the physician and actuary. If their importance were once recognised by the insurance offices, we believe that the necessary data would be readily forthcoming. As illustrations take the following :—

(i) A man's father dies at 40, and his brother at 25. What is the probable reduction in his own life ? *Answer :* 12 years.

(ii) A man has two brothers, who die young at 25 and 29. How much will this shorten his probable duration of life ? *Answer :* 14 years.

(iii) A man's father died at 40, and his brother, his senior by one year, died at 50, twenty-two years ago. A life estate now accrues to the man, whose whereabouts are unknown. What is the probability that he is still alive, and should he return and claim the estate, how long is he likely to enjoy it. *Answer :* The man belongs to an array of men of mean age 54·99 years at death, and standard deviation 16·41 years. Hence the odds against his living beyond 71 years are 835 to 165, and, accordingly, the odds against the possibility of his return are about 21 to 4. Should he be alive and return, he is as likely as not to hold it for 6·8 years, and 8·7 years is his expectancy of life, so that the contingency of his being alive and enjoying the estate is worth only about 1·4 years' income of the estate.

Clearly such problems can be extended in a great variety of ways, which might be serviceable in actuarial practice.

I.—Correlation Table for the Inheritance of Longevity from Father to Son.

First Series.

Age of father at death.

Age of son at death.

20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100	105	Totals.
25	1		1	3	1	1	9	4	3	5	4	3	1	1				37
30	1		3	1	4	1	6	2	6	3	10	5	2	1				45
35			1	2	4	6	4	5	10	6	6	5	7		1			57
40		1	5	2	3	5	6	6	11	7	17	11	2	1				77
45		1	2	2	3	3	5	6	11	11	8	6	4	1				63
50	1	2	2	1	3	7	13	10	4	8	14	11	7	1				84
55	1	2	3	5	5	8	10	7	6	12	9	8	5	2				83
60	1	1		2	2	6	9	6	7	17	13	10	2	2	1			79
65		1	2	5	5	4	11	10	5	18	18	14	10	4				107
70		2	3	1	4	10	9	12	23	9	20	12	10	3	1			119
75	3	1	2	4	3	9	6	10	8	20	23	13	10	6	2			120
80	1		3	1	3	2	6	3	6	9	10	6	8	1				59
85				1	2	1	3	2	6	6	10	8	8	2	1		1	51
90			1		2		2	2		2	2	3						14
95									2		1		1					4
100																		0
105						1												1
Totals	9	11	28	30	44	64	99	85	108	133	165	115	77	25	6	0	1	1000

II.—Correlation Table for the Inheritance of Longevity from Father to Son.

Second Series.

Age of father at death.

Age of son at death.	20 to 25	25 to 30	30 to 35	35 to 40	40 to 45	45 to 50	50 to 55	55 to 60	60 to 65	65 to 70	70 to 75	75 to 80	80 to 85	85 to 90	90 to 95	95 to 100	100 to 105	Totals.
20 to 25	—	—	—	1·5	3·0	2·0	3·5	5·25	1·25	4·5	6·5	4·0	2·0	—	—	—	—	33·5
25 to 30	—	—	—	2·0	3·0	1·75	4·25	4·25	1·75	4·25	4·75	4·5	1·0	—	—	—	—	31·5
30 to 35	1	—	—	1·5	1·0	1·75	5·75	1·0	2·5	5·25	3·75	4·0	4·25	1·75	—	—	—	33·5
35 to 40	—	0·25	2·25	3·0	1·0	1·5	1·5	7·0	5·0	6·5	1·5	7·0	2·75	2·25	1·0	—	—	42·5
40 to 45	—	0·25	0·75	1·5	4·5	2·5	9·5	5·75	8·25	6·0	5·0	3·0	1·0	1·5	1·0	—	—	50·5
45 to 50	—	2·0	0·5	—	1·0	2·5	9·25	4·5	10·0	3·75	8·0	6·25	6·0	5·75	0·5	—	—	60·0
50 to 55	—	—	—	4·25	7·75	6·5	10·75	7·75	10·75	12·25	7·5	11·5	5·25	0·75	—	—	1	86·0
55 to 60	—	—	—	3·25	3·75	3·25	9·5	10·25	9·5	10·5	11·75	17·0	5·75	4·5	0·5	—	—	89·5
60 to 65	—	1·0	3·0	2·5	2·5	4·75	3·5	10·25	11·5	12·5	12·75	19·75	5·75	3·0	1·25	—	—	94·0
65 to 70	—	2·25	1·25	3·0	6·0	4·0	7·0	12·0	12·5	22·75	18·0	20·25	10·75	8·25	4·75	0·25	—	133·0
70 to 75	—	1·25	1·25	0·5	2·75	4·75	5·0	10·5	16·25	13·75	10·75	23·25	14·25	7·75	3·75	0·75	1	117·5
75 to 80	—	—	2·0	1·5	4·75	11·5	8·25	8·0	20·25	11·5	12·5	12·25	13·0	10·25	2·75	1·0	—	119·5
80 to 85	—	0·5	1·0	3·5	1·0	3·25	5·25	8·0	6·5	7·0	7·5	7·0	2·75	3·75	2·0	—	—	59·0
85 to 90	—	—	—	0·5	2·0	1·5	1·5	0·5	3·0	3·5	4·75	6·5	6·75	6·0	—	1·0	—	37·5
90 to 95	—	—	—	0·5	—	—	1·0	1·0	1·0	1·0	1·0	0·75	2·75	1·0	—	—	—	10·0
95 to 100	—	—	—	—	—	—	—	—	—	—	1·5	—	—	—	—	—	—	1·5
100 to 105	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1·0	—	—	1·0
Totals...	1	7·5	12·0	29·0	44·0	51·5	85·5	96·0	120·0	125·0	117·5	147·0	84·0	56·5	18·5	3·0	2	1000·0

III.—Correlation Table for the Inheritance of Longevity in Brethren.

Symmetrical Table, 1000 Cases as 2000.

Age of first brother at death.

Age of second brother at death.	20 to 25	25 to 30	30 to 35	35 to 40	40 to 45	45 to 50	50 to 55	55 to 60	60 to 65	65 to 70	70 to 75	75 to 80	80 to 85	85 to 90	90 to 95	95 to 100	100 to 105	Totals.
20 to 25	—	3·5	3·5	3·0	2·5	3·5	3·5	—	4·5	1·25	3·5	3·0	0·75	0·5	1·0	—	—	34·0
25 to 30	3·5	15·0	5·0	4·5	3·0	3·5	6·0	5·5	3·0	4·75	1·5	5·0	2·75	1·5	—	—	—	64·5
30 to 35	3·5	5·0	11·5	7·75	6·75	6·5	5·75	8·75	4·0	9·0	7·5	6·0	2·5	3·0	0·5	—	—	88·0
35 to 40	3·0	4·5	7·75	7·0	7·0	7·75	11·25	8·25	9·0	12·0	13·0	2·5	3·0	1·0	0·5	—	—	97·5
40 to 45	2·5	3·0	6·75	7·0	10·5	11·25	10·0	7·0	16·0	6·0	14·75	4·75	8·0	1·0	1·0	—	—	109·5
45 to 50	3·5	3·5	6·5	7·75	11·25	20·5	7·75	9·25	12·75	11·25	22·25	15·5	5·5	2·75	—	—	—	140·0
50 to 55	3·5	6·0	5·75	11·25	10·0	7·75	14·0	14·25	17·25	22·25	12·25	12·75	9·0	5·5	0·5	—	—	152·0
55 to 60	—	5·5	8·75	8·25	7·0	9·25	14·25	15·0	19·0	22·5	13·25	25·25	11·5	6·5	1·5	—	—	167·5
60 to 65	4·5	3·0	4·0	9·0	16·0	12·75	17·25	19·0	21·5	32·5	22·25	19·5	17·0	7·75	1·5	—	—	207·5
65 to 70	1·25	4·75	9·0	12·0	6·0	11·25	22·25	22·5	32·5	28·5	31·25	26·25	16·0	13·75	3·75	—	—	241·0
70 to 75	3·5	1·5	7·5	13·0	14·75	22·25	12·25	13·25	22·25	31·25	38·0	27·0	21·75	10·75	1·5	—	—	240·5
75 to 80	3·0	5·0	6·0	2·5	4·75	15·5	12·75	25·25	19·5	26·25	27·0	34·5	19·75	12·0	3·75	—	—	217·5
80 to 85	0·75	2·75	2·5	3·0	8·0	5·5	9·0	11·5	17·0	16·0	21·75	19·75	14·5	5·5	2·0	—	—	139·5
85 to 90	0·5	1·5	3·0	1·0	1·0	2·75	5·5	6·5	7·75	13·75	10·75	12·0	5·5	8·0	2·0	—	—	81·5
90 to 95	1·0	—	0·5	0·5	1·0	—	0·5	1·5	1·5	3·75	1·5	3·75	2·0	2·0	—	—	—	19·5
95 to 100	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
100 to 105	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
Totals	34·0	64·5	88·0	97·5	109·5	140·0	152·0	167·5	207·5	241·0	240·5	217·5	139·5	81·5	19·5	—	—	2000·0